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Supergravity S-Branes

Martin Kruczenski[ab], **Robert C. Myers**[bc] **and Amanda W. Peet**[a]

[a] *Department of Physics, University of Toronto*
St. George Street, Toronto, Ontario M5S 1A7, Canada
[b] *Perimeter Institute for Theoretical Physics*
King Street North, Waterloo, Ontario N2J 2W9, Canada
[c] *Department of Physics, University of Waterloo*
Waterloo, Ontario N2L 3G1, Canada, and
Department of Physics, McGill University
Montréal, Québec H3A 2T8, Canada
E-mail: martink@physics.utoronto.ca, rcm@hep.physics.mcgill.ca
peet@physics.utoronto.ca

Abstract: We construct supergravity solutions corresponding to space-like branes in string theory. Our approach is to apply the usual solution generating techniques to an appropriate time-dependent solution of the eleven dimensional vacuum Einstein equations. In this way all SDp-brane solutions are obtained, as well as the NS and M-theory space branes. Bound states of SDp/SD$(p-2)$- and SDp/SD$(p-4)$-branes are also constructed. Finally, we begin an investigation of the near-brane regions and singularity structure of these solutions.

Keywords: M-Theory, p-branes, D-branes.

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1. Introduction

Recently space-like (and null) branes and their role in superstring theory was discussed by Gutperle and Strominger [1]. These investigations were motivated by the suggestion that such space-branes may lead to a holographic duality which reconstructs a time-like direction, just as investigations of (time-like) D-branes lead to a space-like holography in the form of the AdS/CFT correspondence [2]. Such a time-like holography has been argued to exist in the context of the de Sitter space [3, 4] and may play a role in understanding cosmological backgrounds with the framework of string theory.

Our understanding of the ordinary (time-like) D-branes has developed in recent years with a wide variety of complementary descriptions. Within perturbative string theory, D-branes can be defined as surfaces where open strings end. However, they also have alternative descriptions as supergravity solutions or as solitons in the tachyonic field theory on brane-antibrane systems. In [1], applications of all these descriptions were discussed for space-like branes. In particular, it was suggested that from the open string point of view, they correspond to imposing Dirichlet boundary conditions in the time direction. In brane-antibrane systems they appear as time-dependent tachyon configurations. In supergravity they correspond to solutions describing an incoming spherical wave packet which at later times expands again as a spherical wave.

While much work remains to be done on the perturbative description of space-branes, the understanding of these objects from the point of view of low energy supergravity will be central to determining their holographic properties. Unfortunately, only two supergravity solutions corresponding to S-branes were presented in [1]. The first corresponded to an S0-brane solution of the four-dimensional Einstein-Maxwell theory. The second solution corresponds to the SM5-brane solution of eleven-dimensional supergravity. The main focus of this paper is to expand the repertoire of supergravity S-brane solutions. In particular, we will show that the solution-generating techniques that have been developed within string theory can be equally well applied to this problem. In fact, we will be able to find solutions corresponding to all SDp-branes by applying various duality transformations to a family of solutions of Einstein's equations in eleven dimensions. We note that similar time-dependent solutions also appear in [5], but these represent a special case of the S-branes constructed here. The latter also seem to be closely related to earlier cosmological solutions considered in [6]. We should add that our construction provides solutions of the standard type II supergravities, and so these are distinct from the type II* E-branes considered in [4, 7].

The remainder of the paper is organized as follows: in section 2, we describe our approach of using standard duality transformations to construct supergravity solutions corresponding to SDp-branes with $p \leq 6$, as well as Neveu-Schwarz and M-theory space-branes. In section 3, we investigate the far and near brane behaviour of the solutions. In particular, we find certain solutions for which the scalar curvature is finite in the near brane region. Section 4 presents a discussion of our new solutions. In appendix A, we construct multiply charged solutions corresponding to SDp/SD$(p-2)$- and SDp/SD$(p-4)$-branes. We also find an embedding for the S0-brane solution of [1] in type-IIA string theory as an SD6/SD0-brane. Finally in appendix B, we construct a type-IIB supergravity solution corresponding to an SD(-1)-brane. Throughout the paper, we use the notation of [1] where an Sp-brane has a spatial worldvolume of $p+1$ dimensions.

2. Construction

Our strategy will be to apply the usual solution generating techniques which arise in string theory [8]. With these tools, the problem of solving for new solutions is reduced to an algebraic one, rather than having to resort to solving the nonlinear differential equations provided by Einstein's equations. We will start with an eleven-dimensional solution of the vacuum Einstein equations possessing the appropriate symmetries and then perform a *rotation* mixing the eleventh dimension with one of space-like dimensions. Then dimensionally reducing on the eleventh dimension produces an SD0-brane solution of type-IIA supergravity, smeared in some number of transverse directions. Then applying T-duality transformations on the latter directions gives the desired SDp-brane solutions. Further lifts and S-duality transformations also allow us to construct S-brane versions of the M2- and M5-branes in M-theory and those for the fundamental string and NS5-branes.

However, in order to gain some intuition for the subsequent solutions, we begin by presenting the following metrics on eleven-dimensional flat space:

$$ds^2 = \left(-dt^2 + t^2 dH^2_{8-p}\right) + \sum_{i=1}^{p+2} \left(dx^i\right)^2 \,, \tag{2.1}$$

$$ds^2 = \left(dr^2 + r^2 d\Sigma^2_{8-p}\right) + \sum_{i=1}^{p+2} \left(dx^i\right)^2 \,. \tag{2.2}$$

Above, dH^2_{8-p} and $d\Sigma^2_{8-p}$ represent the line element on the $(8-p)$-dimensional hyperbolic and de Sitter spaces with unit curvature, respectively — we assume here and in the following that $p \leq 6$. Note that in both of these flat-space metrics, translation and rotation invariance is manifest in the $p{+}1$ x^i directions. As well, there is $\mathrm{SO}(1, 8-p)$ Lorentz symmetry on either dH^2_{8-p} or $d\Sigma^2_{8-p}$. Both of these are symmetries which might be desired for an Sp-brane [1]. If we consider the $(9-p)$-dimensional subspace at, say, $x^i = 0$, we see that it is actually Minkowski space with unusual coordinates. In the first case (2.1), the inside of the light-cone[1] (which appears at $t = 0$) is foliated by hyperbolic surfaces with curvature $-1/t^2$. Flat space with these coordinates is sometimes referred to as the Milne universe. In the second metric (2.2), the region outside of the light-cone (which appears at $r = 0$) is foliated by de Sitter space slices with curvature $1/r^2$. This structure is illustrated in figure 1. Our S-brane solutions will have similar coordinates but, of course, the metric components will have extra dependences on t or r. However, these new solutions will approach the above flat-space metrics asymptotically as $t, r \to \infty$.

To begin the construction described above, we must find an appropriate solution of the eleven-dimensional Einstein equations:

$$ds^2 = \beta^G \alpha^H \left(-dt^2 + t^2 dH^2_{8-p}\right) + \left(\frac{\beta}{\alpha}\right)^{\tilde{k}} dz^2 + \sum_{i=1}^{p+1} \left(\frac{\beta}{\alpha}\right)^{k_i} \left(dx^i\right)^2 \tag{2.3}$$

where

$$\alpha = 1 + \left(\frac{\omega}{t}\right)^{7-p} \,, \qquad \beta = 1 - \left(\frac{\omega}{t}\right)^{7-p} \,. \tag{2.4}$$

We will assume $\omega > 0$ throughout the following. Further we have singled out the coordinate z anticipating that we will do a dimensional reduction to ten dimensions on this direction. Solving $R_{AB} = 0$ constrains the exponents to satisfy

$$\begin{aligned} \tilde{k}^2 + \sum_{i=1}^{p+1} k_i^2 + \frac{7-p}{4}(H-G)^2 - 4\frac{8-p}{7-p} &= 0 \,, \\ \tilde{k} + \sum_{i=1}^{p+1} k_i - \frac{7-p}{2}(H-G) &= 0 \,, \\ H + G - \frac{4}{7-p} &= 0 \,. \end{aligned} \tag{2.5}$$

[1]There are actually two patches with identical metrics corresponding to the past and future light-cones.

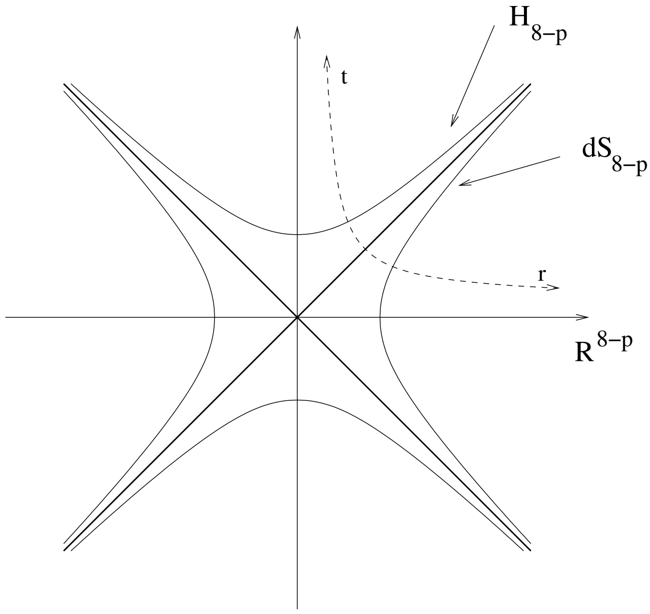

Figure 1: Flat space parametrization appropriate for the construction described in the text.

This solution was found by the liberal application of 'Wick rotations' to the general solutions presented in [9]. Note that most of these solutions are singular at $t = \omega$. One exception, however, is the special case

$$\tilde{k} = 2\,, \qquad H = \frac{4}{7-p}\,, \qquad G = k_i = 0\,, \tag{2.6}$$

for which $t = \omega$ is a horizon, and the corresponding Penrose diagram takes the form shown in figure 2. See section 4 for further discussion.

Similarly, there is a family of 'exterior' solutions

$$ds^2 = \beta^G \alpha^H \left(dr^2 + r^2 d\Sigma^2_{8-p}\right) + \left(\frac{\beta}{\alpha}\right)^{\tilde{k}} dz^2 + \sum_{i=1}^{p+1} \left(\frac{\beta}{\alpha}\right)^{k_i} \left(dx^i\right)^2 \tag{2.7}$$

where

$$\alpha = 1 + \left(\frac{\omega}{r}\right)^{7-p} \qquad \beta = 1 - \left(\frac{\omega}{r}\right)^{7-p} \tag{2.8}$$

and the exponents are constrained as in eq. (2.5). For the special case (2.6) (and a specific periodicity for z — see section 4), these solutions become higher dimensional generalizations of Witten's Kaluza-Klein bubble [10]. In the following, we will focus on the time-dependent solutions (2.3), however, all of these constructions follow through in the same way beginning with these solutions (2.7).

2.1 The IIA SD0-brane

Our basic construction was outlined in the discussion above. We explicitly illustrate the calculations here in the construction of the SD0-brane solution of type-IIA supergravity.

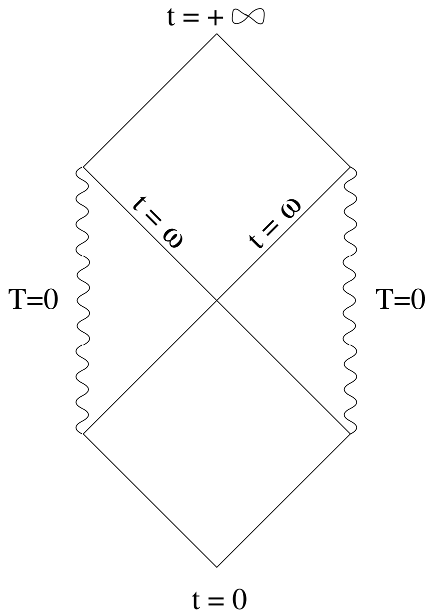

Figure 2: The Penrose diagram for the solutions (2.3) with the special choice of exponents given in eq. (2.6). Two asymptotically flat regions (at $t \to \infty$ and $t \to 0$) are joined by a throat. The horizons (at $t = \omega$, $z = \pm\infty$) in the throat region surround time-like singularities (at $T \to 0$).

We begin with the above solution (2.3) with $p = 0$. The strategy is to then to apply a rotation on the two coordinates, z and x^1:

$$\tilde{x}^1 = \cos\theta\, x^1 - \sin\theta\, z \qquad \tilde{z} = \cos\theta\, z + \sin\theta\, x^1 \tag{2.9}$$

with some fixed angle θ. Next follow this with the standard Kaluza-Klein compactification on the $\tilde{z}$ direction, from eleven-dimensional supergravity to ten-dimensional type-IIA supergravity:

$$ds^2 = e^{-2\Phi/3} G_{\mu\nu} dx^\mu dx^\nu + e^{4\Phi/3}\left(d\tilde{z} + C^{(1)}_\mu dx^\mu\right)^2 \tag{2.10}$$

where $G_{\mu\nu}$ is the ten-dimensional string-frame metric. We note for the present case that

$$\left(\frac{\beta}{\alpha}\right)^{\tilde{k}} dz^2 + \left(\frac{\beta}{\alpha}\right)^{k_1} \left(dx^1\right)^2 = \frac{1}{F}\left(\frac{\beta}{\alpha}\right)^{k_1+\tilde{k}} (d\tilde{x}^1)^2 + F\left(d\tilde{z} + \sin\theta\cos\theta\,\frac{C}{F}\, d\tilde{x}^1\right)^2 \tag{2.11}$$

where we have introduced

$$\begin{aligned} C(t) &= \left(\frac{\beta}{\alpha}\right)^{k_1} - \left(\frac{\beta}{\alpha}\right)^{\tilde{k}}, \\ F(t) &= \cos^2\theta \left(\frac{\beta}{\alpha}\right)^{\tilde{k}} + \sin^2\theta \left(\frac{\beta}{\alpha}\right)^{k_1}. \end{aligned} \tag{2.12}$$

The final type-IIA solution corresponding to an SD0-brane is then

$$\begin{aligned} ds^2 &= F^{\frac{1}{2}} \beta^G \alpha^H \left(-dt^2 + t^2 dH_8^2\right) + F^{-\frac{1}{2}} \left(\frac{\beta}{\alpha}\right)^{k_1+\tilde{k}} \left(dx^1\right)^2 , \\ \exp(2\Phi) &= F^{\frac{3}{2}} , \\ C^{(1)} &= \sin\theta \cos\theta \, \frac{C}{F} \, dx^1 , \end{aligned} \tag{2.13}$$

where we replaced $\tilde{x}^1 \to x^1$. Note that in contrast to the usual D-brane constructions, there is no extremal limit here, i.e., one can not take the limit $\omega \to 0$ while $\cos\theta$, $\sin\theta \to \infty$, since the latter are trigonometric functions (as opposed to the hyperbolic functions which appear in constructing the usual Dp-brane solutions). In this case, the exponents (2.6) give a solution which is obviously regular at $t = \omega$.

2.2 SDp-brane solutions

Our starting point is now eqs. (2.3) and (2.4) with general $p+1$. We do the same rotation (2.9) and compactification (2.10), as above. The resulting type-IIA solution is now

$$\begin{aligned} ds^2 &= F^{\frac{1}{2}} \left[\beta^G \alpha^H \left(-dt^2 + t^2 dH_8^2\right) + \sum_{i=2}^{p+1} \left(\frac{\beta}{\alpha}\right)^{k_i} \left(dx^i\right)^2\right] + F^{-\frac{1}{2}} \left(\frac{\beta}{\alpha}\right)^{k_1+\tilde{k}} \left(dx^1\right)^2 , \\ \exp(2\Phi) &= F^{\frac{3}{2}} , \\ C^{(1)} &= \sin\theta \cos\theta \, \frac{C}{F} \, dx^1 , \end{aligned} \tag{2.14}$$

where $C(t)$ and $F(t)$ are defined as in eq. (2.12), in terms of the α and β given in eq. (2.4). This solution can be regarded as an SD0-brane smeared out over the p directions x^i with $i = 2, \ldots, p+1$. The SDp-brane solution is produced by applying the usual T-duality transformations [11] on the latter p coordinates. The final result is

$$\begin{aligned} ds^2 &= F^{\frac{1}{2}} \beta^G \alpha^H \left(-dt^2 + t^2 dH_{8-p}^2\right) + F^{-\frac{1}{2}} \left[\sum_{i=2}^{p+1} \left(\frac{\beta}{\alpha}\right)^{-k_i} \left(dx^i\right)^2 + \left(\frac{\beta}{\alpha}\right)^{k_1+\tilde{k}} \left(dx^1\right)^2\right] , \\ \exp(2\Phi) &= F^{\frac{3-p}{2}} \left(\frac{\beta}{\alpha}\right)^{-\sum_{i=2}^{p+1} k_i} , \\ C^{(p+1)} &= \sin\theta \cos\theta \, \frac{C}{F} \, dx^1 \wedge \cdots \wedge dx^{p+1} . \end{aligned} \tag{2.15}$$

Note that the general metric above is not isotropic in the worldvolume directions i.e., x^i with $i = 1, \ldots, p+1$. However, from a microscopic point of view, one might expect that the supergravity solution corresponding to a SD-brane would have an isotropic worldvolume as there is nothing to distinguish the various directions at this level. Isotropy in the worldvolume will be restored in the above solution if one chooses $-k_2 = \cdots = -k_{p+1} = k_1 + \tilde{k} \equiv n$. Notice that this excludes the case (2.6). If we also define $k_1 - \tilde{k} \equiv m$, the constraints (2.5) on the exponents in this isotropic case reduce to:

$$\begin{aligned} H &= \frac{2-(p-1)n}{7-p} , \qquad G = \frac{2+(p-1)n}{7-p} , \\ 9(p+1)n^2 + (7-p)m^2 &= 8(8-p) . \end{aligned} \tag{2.16}$$

p	$n_{\max} = \sqrt{\frac{8(8-p)}{9(p+1)}}$	$m_{\max} = \sqrt{8\frac{8-p}{7-p}}$
0	$\frac{8}{3}$	$\frac{8}{\sqrt{7}}$
1	$\frac{2}{3}\sqrt{7}$	$2\sqrt{\frac{7}{3}}$
2	$\frac{4}{3}$	$4\sqrt{\frac{3}{5}}$
3	$\frac{\sqrt{10}}{3}$	$\sqrt{10}$
4	$\frac{4}{3}\sqrt{\frac{2}{5}}$	$4\sqrt{\frac{2}{3}}$
5	$\frac{2}{3}$	$2\sqrt{3}$
6	$\frac{4}{21}\sqrt{7}$	4

Table 1: Maximum values of the parameters n and m for an isotropic SDp-brane.

The corresponding metric simplifies to

$$ds^2 = F^{\frac{1}{2}}(\beta\alpha)^{\frac{2}{7-p}}\left(\frac{\beta}{\alpha}\right)^{n\frac{p-1}{7-p}}\left(-dt^2 + t^2 dH^2_{8-p}\right) + F^{-\frac{1}{2}}\left(\frac{\beta}{\alpha}\right)^{n}\sum_{i=1}^{p+1}\left(dx^i\right)^2 , \tag{2.17}$$

while the dilaton becomes $\exp(2\Phi) = F^{\frac{3-p}{2}}(\beta/\alpha)^{pn}$. Notice that m only appears implicitly[2] above in F through the definition given in eq. (2.12). The maximum values that n and m can have for every dimension $p = 0, \ldots, 6$ are displayed in table 1.

2.3 Neveu-Schwarz S-branes

Given the SDp-brane solutions above, we can construct solutions corresponding to the fundamental string and the NS5-brane, by using S-duality in the type-IIB theory. Since only Neveu-Schwarz fields are excited in these solutions, they will also be solutions for the type-IIA, heterotic, type-I and bosonic string theories.

To construct the solution for a space-like fundamental string, we begin with the SD1-brane of the type-IIB theory. This is given by substituting $p = 1$ into the general solution (2.15) of the previous section:

$$\begin{aligned} ds^2 &= F^{\frac{1}{2}}\beta^G\alpha^H\left(-dt^2 + t^2 dH_7^2\right) + F^{-\frac{1}{2}}\left[\left(\frac{\beta}{\alpha}\right)^{-k_2}\left(dx^2\right)^2 + \left(\frac{\beta}{\alpha}\right)^{k_1+\tilde{k}}\left(dx^1\right)^2\right], \\ \exp(2\Phi) &= F\left(\frac{\beta}{\alpha}\right)^{-k_2}, \\ C^{(2)} &= \sin\theta\cos\theta\,\frac{C}{F}\,dx^1\wedge dx^2 . \end{aligned} \tag{2.18}$$

[2]Therefore, we have a two-parameter family of isotropic solutions, parametrized by (ω, n).

Now it is straightforward to perform an S-duality transformation to obtain the desired SF-string. S-duality maps [12]: $\Phi \to -\Phi$, $G_{\mu\nu} \to e^{-\Phi} G_{\mu\nu}$ and $C^{(2)} \to B^{(2)}$. The result is

$$\begin{aligned}
ds^2 &= \beta^G \alpha^H \left(\frac{\beta}{\alpha}\right)^{\frac{k_2}{2}} \left(-dt^2 + t^2 dH_7^2\right) + \\
&\quad + F^{-1} \left[\left(\frac{\beta}{\alpha}\right)^{-\frac{k_2}{2}} \left(dx^2\right)^2 + \left(\frac{\beta}{\alpha}\right)^{k_1+\tilde{k}+\frac{k_2}{2}} \left(dx^1\right)^2\right], \\
\exp(2\Phi) &= F^{-1} \left(\frac{\beta}{\alpha}\right)^{k_2}, \\
B^{(2)} &= \sin\theta \cos\theta \, \frac{C}{F} \, dx^1 \wedge dx^2 \,.
\end{aligned} \tag{2.19}$$

Note that the solution becomes isotropic in the x^1 and x^2 directions if $-k_2 = k_1 + \tilde{k} = n$. In this case, if we again define $k_1 - \tilde{k} = m$, the constraints (2.5) on the exponents yield:

$$H = G = \frac{1}{3} \qquad\qquad 3n^2 + m^2 = \frac{28}{3} \,. \tag{2.20}$$

Notice here the case (2.6) is also excluded.

The SNS5-brane follows from S-dualizing the SD5-brane solution, which from eq. (2.15) can be written:

$$\begin{aligned}
ds^2 &= F^{\frac{1}{2}} \beta^G \alpha^H \left(-dt^2 + t^2 dH_3^2\right) + F^{-\frac{1}{2}} \left[\sum_{i=2}^{6} \left(\frac{\beta}{\alpha}\right)^{-k_i} \left(dx^i\right)^2 + \left(\frac{\beta}{\alpha}\right)^{k_1+\tilde{k}} \left(dx^1\right)^2\right] \\
e^{2\Phi} &= F^{-1} \left(\frac{\beta}{\alpha}\right)^{-\bar{k}} \\
F^{(3)} &= 4(k_1 - \tilde{k}) \sin\theta \cos\theta \, \omega^2 \, \varepsilon(H_3)
\end{aligned} \tag{2.21}$$

where $\bar{k} = \sum_{i=2}^{6} k_i$ and $\varepsilon(H_3)$ denotes the volume three-form on the hyperbolic plane H_3 with unit curvature. Rather than presenting the six-form RR potential $C^{(6)}$ as in eq. (2.15), we have presented the field strength of the dual two-form potential, i.e., $F^{(3)} = dC^{(2)} = *dC^{(6)}$. The S-duality transformation then yields the SNS5-brane:

$$\begin{aligned}
ds^2 &= F \left(\frac{\beta}{\alpha}\right)^{\frac{1}{2}\bar{k}} \beta^G \alpha^H \left(-dt^2 + t^2 dH_3^2\right) + \left[\sum_{i=2}^{6} \left(\frac{\beta}{\alpha}\right)^{\frac{1}{2}\bar{k}-k_i} \left(dx^i\right)^2 + \left(\frac{\beta}{\alpha}\right)^{k_1+\tilde{k}+\frac{1}{2}\bar{k}} \left(dx^1\right)^2\right] \\
e^{2\Phi} &= F \left(\frac{\beta}{\alpha}\right)^{\bar{k}} \\
H^{(3)} &= 4(k_1 - \tilde{k}) \sin\theta \cos\theta \, \omega^2 \, \varepsilon(H_3)
\end{aligned} \tag{2.22}$$

where $H^{(3)} = dB^{(2)}$ is the NS three-form field strength. In this case, demanding isotropy in the worldvolume directions produces the same constraints on the exponents as for the SD5-brane. The latter are given by eq. (2.16) with $p = 5$.

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As mentioned above, since no RR fields appear in the solutions (2.19) and (2.22), they will be equally valid as low energy solutions of the type-IIB or IIA, heterotic, type I or even bosonic string theories. One could also generalize the S-duality transformation above to a general SL$(2, R)$ mapping [12], which produce space-branes with both Neveu-Schwarz and RR fluxes. So in particular, eq. (2.19) would be generalized to a space-like (p, q)-string.

2.4 M-theory S-branes

In M-theory there are also space-like counterparts to the usual M2- and M5-branes. These can be obtained by lifting the solutions for the space-like fundamental string (2.19) and the SD4-brane from eq. (2.15) to eleven dimensions.

Thus the SM2-brane is given by:

$$\begin{aligned} ds^2 &= F^{\frac{1}{3}} \beta^G \alpha^H \left(\frac{\beta}{\alpha}\right)^{\frac{k_2}{6}} \left(-dt^2 + t^2 dH_7^2\right) + \\ &\quad + F^{-\frac{2}{3}} \left[\left(\frac{\beta}{\alpha}\right)^{-\frac{5}{6}k_2} \left(dx^2\right)^2 + \left(\frac{\beta}{\alpha}\right)^{k_1+\tilde{k}+\frac{k_2}{6}} \left(dx^1\right)^2 + \left(\frac{\beta}{\alpha}\right)^{\frac{2k_2}{3}} dz^2\right], \\ A^{(3)} &= \sin\theta\cos\theta \frac{C}{F} dx^1 \wedge dx^2 \wedge dz\,, \end{aligned} \tag{2.23}$$

where z denotes the eleventh dimension. Here C and F are again defined as in eq. (2.12), and

$$\alpha = 1 + \left(\frac{\omega}{t}\right)^6, \qquad \beta = 1 - \left(\frac{\omega}{t}\right)^6. \tag{2.24}$$

The exponents are constrained as in eq. (2.5) with $p = 1$. Hence the unique choice for an isotropic brane is $G = H = 1/3$, $k_1 = -\tilde{k} = \pm\sqrt{7/3}$ and $k_2 = 0$, in which case the metric becomes

$$ds^2 = F^{\frac{1}{3}} (\beta\alpha)^{\frac{1}{3}} \left(-dt^2 + t^2 dH_7^2\right) + F^{-\frac{2}{3}} \left[\left(dx^1\right)^2 + \left(dx^2\right)^2 + dz^2\right]. \tag{2.25}$$

The case (2.6) gives rise to an anisotropic brane but it is interesting to note that, in this case, dimensional reduction on x^1 gives rise to an isotropic IIA Sf-string in ten dimensions. While the ten-dimensional metric is singular but it seems that the lift to eleven dimensions may evade the singularity at $t = \omega$ [13] — we consider this point in more detail in section 4.

Similarly the SM5-brane follows from lifting the SD4-brane solution to eleven dimensions:

$$\begin{aligned} ds^2 &= F^{\frac{2}{3}} \beta^G \alpha^H \left(\frac{\beta}{\alpha}\right)^{\frac{\bar{k}}{3}} \left(-dt^2 + t^2 dH_4^2\right) + \\ &\quad + F^{-\frac{1}{3}} \left[\sum_{i=2}^{5} \left(\frac{\beta}{\alpha}\right)^{\frac{\bar{k}}{3}-k_i} \left(dx^i\right)^2 + \left(\frac{\beta}{\alpha}\right)^{k_1+\tilde{k}+\frac{\bar{k}}{3}} \left(dx^1\right)^2 + \left(\frac{\beta}{\alpha}\right)^{-\frac{2}{3}\bar{k}} dz^2\right], \\ F^{(4)} &= -6 \sin\theta\cos\theta\, (k_1 - \tilde{k})\, \omega^3\ \varepsilon(H_4) \end{aligned} \tag{2.26}$$

where we defined $\bar{k} = \sum_{i=2}^{5} k_i$. Also $\varepsilon(H_4)$ denotes the volume form of the four-dimensional hyperbolic plane, i.e., dH_4^2. Choosing $k_{2,3,4,5} = 0$, $k_1 = -\tilde{k} = \pm\sqrt{8/3}$, $G = H = 2/3$

produces an isotropic brane. In this case, the metric simplifies to

$$ds^2 = F^{\frac{2}{3}} (\alpha\beta)^{\frac{2}{3}} \left(-dt^2 + t^2 dH_4^2\right) + F^{-\frac{1}{3}} dx_{(6)}^2 \tag{2.27}$$

where $dx_{(6)}^2$ denotes the metric for flat six-dimensional euclidean space. This isotropic solution should be the same as the SM5-brane found in [1]. In fact it can be shown that the above solution takes the form given in [1] after the change of coordinates $t^{-3} = \tanh(3\tilde{t}/2)$. We also observe that as for the SM2-brane, considering the case (2.6) and dimensionally reducing in x^1 gives rise to an isotropic SD4-brane.

3. Far and near brane limits

It is interesting to study the structure of the SDp-brane solutions[3] in the asymptotic ($t \to \infty$) and near-brane ($t \to \omega$) regions. We will address each of these in turn. In doing so, however, we will only consider the isotropic case where the exponents satisfy the constraints given in eq. (2.16). Hence in the following, recall that for these isotropic S-branes, we have $n = k_1 + \tilde{k}$ and $m = k_1 - \tilde{k}$.

3.1 Far away

Being far from the brane corresponds to taking $t \to \infty$. In such a limit, the SDp-brane metric becomes the flat metric of (2.1). The corrections are easy to compute. First we have:

$$\begin{aligned}
\frac{\beta}{\alpha} &\simeq 1 - 2\left(\frac{\omega}{t}\right)^{7-p} \\
F &\simeq 1 - (n + m\cos 2\theta)\left(\frac{\omega}{t}\right)^{7-p} \\
C &\simeq 2m\left(\frac{\omega}{t}\right)^{7-p}
\end{aligned} \tag{3.1}$$

Then we can find the metric, dilaton and RR potential:

$$\begin{aligned}
ds^2 &\simeq \left(1 - \left[\frac{3}{2}\frac{p+1}{7-p}n + \frac{m}{2}\cos 2\theta\right]\left(\frac{\omega}{t}\right)^{7-p}\right)(-dt^2 + t^2 dH_{8-p}^2) + \\
&\quad + \left(1 - \left[\frac{3}{2}n - \frac{m}{2}\cos 2\theta\right]\left(\frac{\omega}{t}\right)^{7-p}\right) dx_{(p+1)}^2 \\
\Phi &\simeq -\frac{1}{4}\left[3(p+1)n + m\cos 2\theta\right]\left(\frac{\omega}{t}\right)^{7-p} \\
C^{(p+1)} &= 2m\sin\theta\cos\theta\left(\frac{\omega}{t}\right)^{7-p} dx^1 \wedge \cdots \wedge dx^{p+1}
\end{aligned} \tag{3.2}$$

where in the metric, $dx_{(p+1)}^2$ indicates the line element on flat (p+1)-dimensional euclidean space. Note that the power of t in the RR potential is such that the surfaces of constant t will carry a constant flux, that is, $*F^{(p+2)} \propto \varepsilon(H_{8-p})$. In fact, the latter result applies for the full nonlinear solution, as was explicitly shown for the SD5-brane in eq. (2.21). The above form of the asymptotic solution also indicates that we are using the Feynman propagator, that is, the analytic continuation of the euclidean propagator $1/r^{7-p}$. We will return to this point in the discussion section.

[3] The results are very similar for the SF1-, SNS5-, SM2- and SM5-branes.

3.2 Close up metric

Naively, the flat space metric (2.1) would indicate that we approach the brane, i.e., the tip of the light-cone, in the limit $t \to 0$ (with fixed coordinates on the hyperbolic space). However, it is clear from the solutions (2.15) that this intuition must be modified in the case of the strongly gravitating branes. We see that components of the metric may vanish or diverge at $t = \omega$. Hence the most interesting limit when we get close to the brane is $t \to \omega$ where $\beta \to 0$. In fact given the latter, it is useful to use β as a coordinate instead of t. Using this new coordinate, the metric for the isotropic (SO$(p+1)$-invariant) SDp-branes becomes

$$\begin{aligned} ds^2 = & \sqrt{F(\beta)}\beta^G(2-\beta)^H \frac{\omega^2}{(1-\beta)^{2/(7-p)}} \left[-\frac{d\beta^2}{(7-p)^2(1-\beta)^2} + dH^2_{8-p} \right] \\ & + \frac{1}{\sqrt{F(\beta)}} \left(\frac{\beta}{2-\beta} \right)^n dx^2_{(p+1)} \,. \end{aligned} \tag{3.3}$$

For calculating the curvature at small β, it suffices to take

$$ds^2 \simeq 2^H \omega^2 \sqrt{F(\beta)}\beta^G \left[-\frac{d\beta^2}{(7-p)^2} + dH^2_{8-p} \right] + \frac{1}{\sqrt{F(\beta)}} \left(\frac{\beta}{2} \right)^n dx^2_{(p+1)} \tag{3.4}$$

where

$$F(\beta) = \cos^2\theta \left(\frac{\beta}{2} \right)^{\frac{1}{2}(n-m)} + \sin^2\theta \left(\frac{\beta}{2} \right)^{\frac{1}{2}(n+m)} \,. \tag{3.5}$$

At small β, we find the Ricci scalar to be

$$R \simeq \frac{(7-p)^2}{2^{2+H}\omega^2} \frac{\mathcal{R}_p(n)}{\beta^{2+G}\sqrt{F(\beta)}} \tag{3.6}$$

where eq. (2.16) requires that $(7-p)G = 2 + (p-1)n$ and $(7-p)H = 2 - (p-1)n$. Then, with the abbreviation $\dot{} \equiv \beta(d/d\beta)$, the function $\mathcal{R}_p(n)$ can be written as

$$\begin{aligned} \mathcal{R}_p(n) = & \left(\frac{\ddot{F}}{F} \right) [2(7-2p)] + \left(\frac{\dot{F}}{F} \right)^2 [(p-3)(p+1)] + \left(\frac{\dot{F}}{F} \right) \left[n \left(-4p^2 + 12p - 2 \right) \right] + \\ & + \frac{1}{(7-p)} \left[-4(8-p) + n^2(-4p^3 + 28p^2 + 4p + 8) \right] . \end{aligned} \tag{3.7}$$

Equation (2.16) also fixes m with $(7-p)m^2 = 8(8-p) - 9(p+1)n^2$. Let us assume for definiteness that $m > 0$, i.e., take the positive[4] root for m in the constraint equation. Then by eq. (3.5), the first term in F and its derivatives will dominate at small β. Next, we need to inspect $\mathcal{R}_p(n)$ in the expression for R. All derivatives of F appearing here are of the same order as F itself. In fact, at leading order in β, $\dot{F}/F = (n-m)/2$, and $\ddot{F}/F = (n-m)^2/4$. This gives $\mathcal{R}_p(n)$ as a function of n and p only. For illustrative purposes, we plot this function for both signs of m in figure 3.

[4]For $m < 0$, we simply swap $\cos\theta \leftrightarrow \sin\theta$ and $+m \leftrightarrow -m$ in the following.

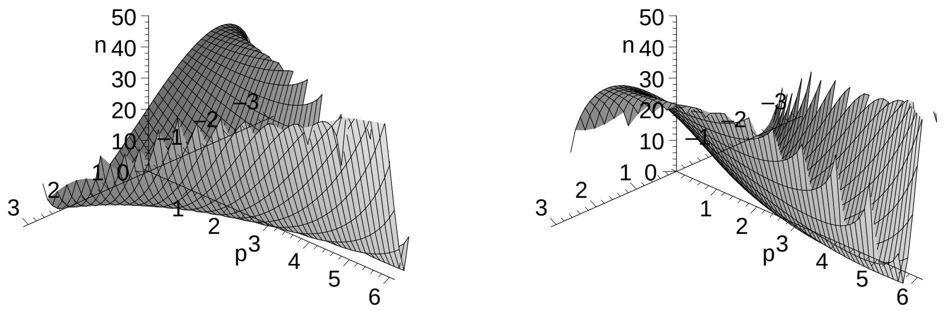

Figure 3: The function $\mathcal{R}_p(n)$, as a function of n, p for $m > 0$ and $m < 0$ respectively. (The edges of the surfaces occur where n runs out due to the reality constraint on m.)

The function $\mathcal{R}_p(n)$ has a zero, so for all p there is a solution with R=0 at $\beta \to 0$. The root of $\mathcal{R}_p(n)$ can be solved for analytically:

$$n_\pm(p) = \pm\frac{(p-3)}{6}\sqrt{\frac{2(8-p)}{(p+1)}} \quad = \pm\frac{(p-3)}{4}\,|n_{\max}| \tag{3.8}$$

where $n_{\max}$ is the maximum allowed value of n, tabulated in table 1. Also, as can be seen from figure 3, the magic value n_- must be chosen for $m > 0$, and n_+ for $m < 0$.

For generic values of n, p, however, $\mathcal{R}_p(n)$ will not vanish. Then, the behaviour of R at small β is governed by

$$R \simeq \frac{1}{\omega^2}\frac{1}{\beta^{2+G}\sqrt{F(\beta)}} \simeq \frac{1}{\omega^2}\frac{1}{\cos\theta}\beta^{-\delta_g} \tag{3.9}$$

where the exponent δ_g is given by

$$\delta_g = \frac{[8(8-p) + 3n(1+p) - m(7-p)]}{4(7-p)} \tag{3.10}$$

By expressing m in terms of n and p via the constraint (2.16), it can be shown that δ_g is strictly positive. This implies that R blows up at $\beta \to 0$ generically.

The SNS-branes and SM-branes can be analyzed similarly. The expression for R is almost identical, except for different powers of F and β in the denominator. δ_g is still strictly positive. We find zeroes in $\mathcal{R}$ as follows: SF1: $n_\pm = \pm\sqrt{7}/3$, SNS5: $n_\pm = \pm 1/3$, SM2: $n_\pm = 0, \pm 4/3$, and SM5: $n_\pm = 0$. Note in particular the root $n = 0$ for the SD3, SM2, and SM5. Also note that for $p = 0$ the root $n = 2$ corresponds to the case (2.6).

To demonstrate that a solution is nonsingular, however, it is not sufficient to show that R=0. It is tedious to check the $R^{\mu\nu}R_{\mu\nu}$ and $R^{\mu\nu\lambda\sigma}R_{\mu\nu\lambda\sigma}$ invariants. We have explicitly computed both, using the full metric, for the $n = 0$ cases (SD3, SM2, SM5). We find that $R^{\mu\nu}R_{\mu\nu}$ is zero at $\beta \to 0$, but that $R^{\mu\nu\lambda\sigma}R_{\mu\nu\lambda\sigma}$ blows up there. Therefore, these solutions are singular, if in a somewhat milder way. This was most likely inevitable due to the fractional powers appearing in the metric, even for $n = 0$.

3.3 Close up dilaton and R-R fields

We can also consider the close-up dilaton behaviour for the SDp-branes. We find that

$$e^{\Phi} \sim (\cos\theta)^{(3-p)/2} \left(\frac{\beta}{2}\right)^{-\delta_\Phi} \tag{3.11}$$

where the exponent δ_Φ is

$$\delta_\Phi = \frac{1}{8}\left[m(3-p) - 3n(p+1)\right] \tag{3.12}$$

Interestingly, δ_Φ is zero for the magic $n_\pm$ of (3.8), for the same choice of sgn(m):

$$\delta_\Phi = 0\,, \quad \begin{cases} n = n_-(p)\,, & m > 0 \\ n = n_+(p)\,, & m < 0 \end{cases} \tag{3.13}$$

For the R-R field of the SDp-branes,

$$C_{1,\ldots,p+1} = \frac{\cos\theta \sin\theta\; C(\beta)}{F(\beta)} = \frac{\cos\theta \sin\theta \left[\beta^m - (2-\beta)^m\right]}{\left[\cos^2\theta (2-\beta)^m + \sin^2\theta \beta^m\right]} \tag{3.14}$$

Again picking $m > 0$ for definiteness, we find:

$$C^{p+1} \simeq -\tan\theta \left[1 - \sec^2\theta \left(\frac{\beta}{2}\right)^m\right] dx^1 \wedge \cdots \wedge dx^{p+1} \tag{3.15}$$

4. Discussion

In this paper, we have constructed a wide variety of space brane solutions for supergravity equations using the standard solution generating techniques available in string theory [8]. In particular, we have obtained maximally symmetric SDp-brane solutions preserving SO$(p+1) \times$ SO$(8-p,1)$. Note that for each value of p, there is a two parameter family of these isotropic solutions. From a microscopic point of view, it is not clear why there should be two parameters rather than just one corresponding to the number of branes. Note that in contrast the isotropic S-branes arising in M-theory have a single parameter. This uniqueness of the isotropic SM-branes could in principle be used to determine a preferred set of solutions for the SDp-branes.

Central to our construction was finding an appropriate family of generating solutions given in eq. (2.3), which correspond to anisotropic solutions of Einstein's equations in eleven dimensions. Nearly all of these solutions contain curvature singularities at $t = \omega$. As a result, most of the resulting S-brane solutions contain at least mild curvature singularities, as discussed in section 3. However, there remains work to be done in understanding the near-brane regions of the full set of S-brane solutions which we have constructed. In any event, the pervasive appearance of singularities, as well as the extra parameter in the isotropic solutions, suggest that not all of these solutions will be relevant for string theory. That is, reasonable physical sources will not be found in string theory for all of the solutions presented in this paper.

A better behaved set of generating solutions arises with the choice of exponents given in eq. (2.6). In this case, $t = \omega$ becomes a horizon, as is easily seen by making the coordinate transformation: $T = \alpha^{2/(7-p)}t$. In this case, the metric reduces to

$$ds^2 = -f^{-1}(T)\, dT^2 + T^2 dH^2_{8-p} + f(T)\, dz^2 + \sum_{i=1}^{p+1} \left(dx^i\right)^2 \tag{4.1}$$

where $f(T) = 1 - 4\left(\omega/T\right)^{7-p}$ and the Penrose diagram takes the form given in figure 2. Hence there are two asymptotically flat regions (at $t \to \infty$ and $t \to 0$) joined by a throat. One also finds time-like singularities (at $T \to 0$) behind the horizons (at $t = \omega$) in the throat region. With these exponents (2.6) in any of the solutions of section 2, one has an anisotropic S-brane which, however, inherits the same casual structure. This special class of solutions corresponds to those presented in [5]. Similar solutions were also considered in [14].

Of course, the above remarks apply to section 2.4 and so choosing eq. (2.6) produces an anisotropic SM2-brane. However, as noted there, one could make a dimensional reduction on the x^1 direction to produce an isotropic IIA SF-string in ten dimensions. Naively then while the ten-dimensional metric is singular, it seems that the lift to eleven dimensions would evade the singularity at $t = \omega$ [13]. However, one must be cautious because implicitly we are choosing x^1 to be periodic. Hence in the eleven-dimensional solution, this circle shrinks to zero size at $t = \omega$ to produce a 'conical' singularity. The near 'horizon' geometry in the t and x^1 directions is essential flat space. In this limit, it may be possible to understand the periodicity of x^1 as orbifolding by a particular boost symmetry [14]–[17]. Hence M-theory may be able to resolve this singularity, however, one must note that the region behind the 'horizon' seems as though it must be problematic since it contains closed time-like curves.

Further progress in identifying the physically relevant supergravity solutions would come from a better understanding of the microphysical, i.e., the perturbative string, picture of the SD-branes. The suggestion of [1] is that S-branes would appear in perturbative string theory by introducing an open string sector with Dirichlet boundary conditions in the direction of time.[5] Thus these strings are confined to a space-like surface at a given instant of time. The worldvolume theory on such an S-brane seems to be an euclidean Yang-Mills theory. The scalar corresponding to transverse displacements in the time direction would have a negative kinetic term. Such theory appears to contain negative norm states, unless they can be eliminated by means of the gauge fixing. A related unusual property of the field theory would be that the Lorentz symmetry SO$(8-p,1)$ of the transverse space becomes the 'R-symmetry' group, which is hence non-compact. The same symmetry is maintained in our supergravity solutions as the isometry group of the hyperbolic space H_{8-p}.

Recall that there is a class of supergravity solutions generated from eq. (4.1) which is somewhat better behaved at $t = \omega$. However, these space branes are anisotropic with the x^1 direction being distinct from the other worldvolume directions, x^i with $i = 2, \ldots, p+1$. It could be that this particular direction is singled out by imposing different boundary

[5] An open string theory on null-branes was investigated by [18].

conditions on the worldvolume fields. In part, we are motivated to make this suggestion by two observations: first, the generating solution (4.1) is roughly a 'Wick rotation' of a black brane solution. In the context of time-like D-branes, such nonextremal supergravity solutions correspond to having the worldvolume theory at finite temperature. The latter can be evaluated by considering a path integral where antiperiodic boundary conditions are imposed on the fermions in the euclidean time direction. Second, as noticed in section 2 (see further discussion below), for the analogous exterior solutions (2.7), the corresponding coordinate closes off smoothly at $t = \omega$, which results in antiperiodic boundary conditions for the supergravity fermions at infinity. Again similar considerations in the context of time-like D-branes (see, e.g., [19]) would dictate that the worldvolume fermions are also antiperiodic. In any event, if this speculation is correct, our supergravity results would indicate that a Scherk-Schwarz compactification of the worldvolume theory is better behaved than the uncompactified theory.

As noted in [1], the perturbative SD-branes would naturally source on-shell closed strings in the surrounding bulk spacetime. However, there is an ambiguity in these long-range bulk fields which can alternatively be seen as an ambiguity in the Green's function or an ambiguity in the boundary conditions for the bulk fields. In examining the asymptotic late time-behaviour in section 3.1, the fields displayed a $1/t^{7-p}$ behavior. The latter would naturally be associated with the Feynman propagator in Minkowski space, which is the analytic continuation of the usual euclidean Green's function. Enforcing causality, on the other hand, would require the use of the retarded and advanced Green's function. The advanced one would describe fine-tuned incoming radiation that creates the space-brane, and the retarded one would describe the outgoing radiation after the brane disappears [1]. The retarded and advanced Green's functions are different from the Feynman propagator and also have different properties in even and odd dimension, e.g., for even dimensions and massless fields, they only have support on the light-cone. From a causal perspective, the Feynman propagator would contain extra homogeneous solutions which neither help to create the brane nor come from its decay. The possible role of these bulk fields in a brane/bulk duality remains mysterious to us. The apparent advantage of using the Feynman propagator, however, is that solutions of any dimensionality appear to be on more of an equal footing.

Moreover it is not clear to us if supergravity solutions corresponding to the retarded plus advanced propagator actually exist. In fact, in [1], it was argued that the creation of the brane can be described as the excitation of the open string tachyon in an unstable brane or in a brane/anti-brane system. This tachyon field then decays to a (possibly new) vacuum generating an outgoing pulse of radiation. We expect that a supergravity solution can be used to describe this process only when a large number of branes is created. However in that case one might expect that the incoming wave collapses to form a black hole. While it may still excite the tachyon field, the latter stages of this process would be hidden behind an event horizon. This picture is reminiscent of the recent discussion presented in [20]. There it was argued that tachyon lumps corresponding to a large number of branes reside inside their own Schwarzschild radius and consequently, can not decay by classical radiation. The preferred process for large N appears to be the creation of open strings living on the branes

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(as this process is enhanced by a factor of N) rather than closed string emission. For a more detailed discussion of this issues we refer the reader to [20]. Here, we only note that this discussion shows that, if the physics of these supergravity solutions is contained in the theory describing open strings with Dirichlet boundary conditions in the time direction, then such theory should contain a rich structure with quite different regimes at small and large N as well as an ambiguity which corresponds to the choice of boundary conditions in supergravity. It is clear that more work is needed in order to properly understand the worldvolume theory and translate its properties into supergravity statements. In the present paper we assumed that the world volume theory can be defined in such a way as to correspond to the Feynman propagator which on the other hand seems natural since the latter is defined just by an analytic continuation from the euclidean one. Note also that the solutions presented in [1] which contain gravity are also of the same type.

It is interesting that other time-dependent backgrounds have recently been studied from a closed string worldsheet point of view using orbifolds [16, 17, 14, 21, 22, 23]. The time-dependence is generated by involving the time direction in one of the orbifolding operations. These studies may provide useful insights to understanding the spectrum of open string theory living on an SD-brane.

Our approach to constructing SDp-brane solutions worked in a straightforward way for $0 \leq p \leq 6$. One might consider extending this family of solutions beyond this range of p. In appendix B, we construct a solution which seems to correspond to an SD(-1)-brane in type-IIB supergravity. Note that this solution is *not* the usual instantonic D(-1)-brane, but rather a time-dependent solution in ten-dimensional Minkowski-signature spacetime. It seems that one can construct solutions for $p = 7$ and 8 using essentially the same procedures as in section 2. The case $p = 8$ follows straight forwardly from the replacement $p = 8$ in all expressions and the case $p = 7$ can be described somewhat loosely speaking as a limiting procedure $p = 7 - \epsilon$ with $\epsilon \to 0$. In light of the previous discussion, the cases $p = 6, 7$ and 8 are particularly interesting because they involve less than three *space-like* transverse dimensions. Hence an incoming shell would not be expected to form a black hole horizon. To finish this discussion, we add that one can not have an SD9-brane in ten-dimensional Minkowski space, i.e., a nine-brane would necessarily fill the time direction.

In this paper, we focused on the time-dependent solutions generated from eq. (2.3), as these naturally seem to describe the solutions sourced by space-branes, i.e., inside the light-cone of the S-brane source. We would also like to consider briefly the solutions that would be generated in the identical fashion using eq. (2.7) as the generating solution. These would seem to describe solutions in a region casually disconnected from the space-branes, i.e., outside the light-cone of the S-brane source. It is not clear what the role of these regions would be in a holographic description of the S-branes. However, as noted above, our supergravity solutions would seem to be related to a perturbative framework using the Feynman propagator for the bulk fields. The latter would naturally introduce homogeneous waves in the region outside the light-cone, and hence perhaps these 'exterior' solutions could naturally be matched on to the S-brane solutions presented above, at least for certain choices of the parameters. Certainly, a much more thorough investigation of the near-brane regions would be needed before such matching of solutions could be accomplished.

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As noted before, the solutions in eq. (2.7) are closely related to Witten's Kaluza-Klein bubble [10], which demonstrates the instability of the KK vacuum (in certain theories). In fact for $k_i = 0$, $\tilde{k} = 2$, $G = 0$ and $H = 4/(7-p)$, these solutions are higher dimensional generalizations of Witten's solution. The z direction smoothly closes off at $r = \omega$ if z has period $2\pi\, 2^{\frac{9-p}{7-p}}\omega/(7-p)$. Of course, in this solution, one could trade z for for one of the x^i as coordinate on the circle that closes off. As all of the constructions of section 2 follow through unchanged if one begins with the solutions (2.7), and in particular the KK bubbles, one would construct 'charged' generalizations of the usual vacuum solutions. These might provide interesting time-dependent string theory backgrounds [24]. Note, however, as these particular 'exterior' solutions close off smoothly, they would certainly not seem amenable to the matching suggested above.

We close with one final comment on the 'exterior' solutions. These solutions realize the $SO(1, 8-p)$ symmetry by introducing a foliation of the transverse space in terms of de Sitter space slices. If we examined the near-brane region, these solutions would naturally take the form of a warped product of $dS_{8-p} \times R^{p+2}$, similar to those discussed in [25]. As de Sitter space seems to arise quite rarely in string or M-theory [26], these solutions deserve further study.

Acknowledgments

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A. Multiply charged S-branes

In the case of time-like D-branes, an important role is played by bound states where branes of different dimensionality bind together to form a single object carrying two (or more) RR charges. A notable example is the D1/D5 system which plays a central role in investigations of black hole microphysics [28] and the AdS/CFT correspondence [2]. Similarly one can consider multiply charged SD-branes although it is not clear if they will play the same essential role. In any case, to stress the simplicity of our approach, we construct the supergravity solutions corresponding to such multiply charged branes and leave their detailed study for future work.

We start with SDp/SD$(p-2)$-brane solutions (with $p \geq 2$). Their construction is a minor extension of the approach given in section 2.2. There we started from the SD0-brane smeared in p transverse directions and performed p T-dualities. In the present case, we stop after two T-dualities and lift to eleven dimensions producing a smeared SM2-brane.

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Then we rotate the coordinates x^1 and the new eleventh coordinate z by a constant angle γ and dimensionally reduce back producing a smeared SD2/SD0-brane. Finally we perform the last $p-2$ T-dualities. The result is

$$
\begin{aligned}
ds^2 &= \widetilde{F}^{\frac{1}{2}} F^{\frac{1}{2}} \alpha^H \beta^G (-dt^2 + t^2 dH^2_{8-p}) + \widetilde{F}^{\frac{1}{2}} F^{-\frac{1}{2}} \left[\left(\frac{\beta}{\alpha}\right)^{-k_2} (dx^2)^2 + \left(\frac{\beta}{\alpha}\right)^{-k_3} (dx^3)^2 \right] + \\
&\quad + \widetilde{F}^{-\frac{1}{2}} F^{-\frac{1}{2}} \left[\left(\frac{\beta}{\alpha}\right)^{k_1+\tilde{k}} (dx^1)^2 + \sum_{i=4}^{p+1} \left(\frac{\beta}{\alpha}\right)^{-k_i} (dx^i)^2 \right] \\
e^{2\Phi} &= F^{\frac{3-p}{2}} \widetilde{F}^{\frac{5-p}{2}} \left(\frac{\beta}{\alpha}\right)^{-\sum_{i=2}^{p+1} k_i} \\
B^{(2)} &= \sin\gamma \sin\theta \cos\theta \, \frac{C}{F} \, dx^2 \wedge dx^3 \\
C^{(p-1)} &= \sin\gamma \cos\gamma \, \frac{\widetilde{C}}{\widetilde{F}} \, dx^1 \wedge dx^4 \wedge \cdots \wedge dx^{p+1} \\
C^{(p+1)} &= \cos\gamma \sin\theta \cos\theta \, \frac{C}{F} \, dx^1 \wedge \cdots \wedge dx^{p+1}
\end{aligned} \tag{A.1}
$$

where F and C denote the same functions (2.12) as before and we have introduced two new functions $\widetilde{F}$ and $\widetilde{C}$, defined as

$$
\widetilde{F}(t) = \cos^2\gamma + \sin^2\gamma \frac{1}{F} \left(\frac{\beta}{\alpha}\right)^{k_1+\tilde{k}+k_2+k_3} , \qquad \widetilde{C}(t) = \frac{1}{F} \left(\frac{\beta}{\alpha}\right)^{k_1+\tilde{k}+k_2+k_3} - 1 . \tag{A.2}
$$

Further note that α and β are defined as in eq. (2.4) with p (and not $p-2$). As a trivial check we can see that when $\gamma \to 0$, we recover the SDp-brane. Slightly less trivial is to check that for $\theta \to 0$, the solution becomes a smeared SD$(p-2)$-brane. A straightforward calculation shows that this is indeed the case after we make the replacements:

$$
\begin{aligned}
\widehat{H} &= H - q , & \widehat{G} &= G + q , & \widehat{\tilde{k}} &= \tilde{k} - 2q , & \widehat{k}_1 &= k_1 + q , \\
\widehat{k}_2 &= -k_2 - \tilde{k} + q , & \widehat{k}_3 &= -k_3 - \tilde{k} + q , & \widehat{k}_{i=4,\ldots,p+1} &= k_i + q
\end{aligned} \tag{A.3}
$$

with $q = (\tilde{k} + k_3 + k_3)/3$. The 'hatted' values satisfy the conditions (2.5) if the original ones do. Note that these bound state solutions involve a nontrivial Neveu-Schwarz two-form B, just as is found for time-like Dp/D$(p-2)$-brane solutions [29] — see, also, [30].

The SDp/SD$(p-4)$-brane solutions (with $p \geq 4$) are constructed in a similar manner. The final result is

$$
\begin{aligned}
ds^2 &= \widetilde{F}^{\frac{1}{2}} F^{\frac{1}{2}} \left[\beta^G \alpha^H (-dt^2 + t^2 dH_{8-p}) \right] + \widetilde{F}^{\frac{1}{2}} F^{-\frac{1}{2}} \sum_{i=2}^{5} \left(\frac{\beta}{\alpha}\right)^{-k_i} (dx^i)^2 + \\
&\quad + \widetilde{F}^{-\frac{1}{2}} F^{-\frac{1}{2}} \left[\left(\frac{\beta}{\alpha}\right)^{k_1+\tilde{k}} (dx^1)^2 + \sum_{i=6}^{p+1} \left(\frac{\beta}{\alpha}\right)^{-k_i} (dx^i)^2 \right] \\
e^{2\Phi} &= F^{\frac{3-p}{2}} \widetilde{F}^{\frac{7-p}{2}} \left(\frac{\beta}{\alpha}\right)^{-\sum_{i=2}^{p+1} k_i}
\end{aligned}
$$

$$
\begin{aligned}
C^{(p-3)} &= \sin\gamma\cos\gamma\,\frac{\widetilde{C}}{\widetilde{F}}\,dx^1\wedge dx^6\wedge\cdots dx^{p+1}\\
C^{(p+1)} &= \sin\theta\cos\theta\,\frac{C}{F}\,dx^1\wedge\cdots dx^{p+1}
\end{aligned}
\tag{A.4}
$$

In this case the functions $\widetilde{F}$ and $\widetilde{C}(t)$ are defined to be

$$
\widetilde{F}(t) = \cos^2\gamma + \sin^2\gamma\left(\frac{\beta}{\alpha}\right)^{k_1+\tilde{k}+\sum_{i=2}^{5}k_i}\,, \qquad \widetilde{C}(t) = \left(\frac{\beta}{\alpha}\right)^{\tilde{k}+\sum_{i=1}^{5}k_i} - 1\,. \tag{A.5}
$$

Again the $\gamma\to 0$ limit is trivial and one can easily verify that the limit $\theta\to 0$ produces a smeared SD$(p-4)$ solution with the replacements:

$$
\begin{aligned}
\widehat{H} &= H-q, \qquad \widehat{G} = G+q, \qquad \widehat{\tilde{k}} = \tilde{k}-2q, \qquad \widehat{k}_1 = k_1+q,\\
\widehat{k}_{i=2,\ldots,5} &= -k_i-\tilde{k}+q, \qquad \widehat{k}_{i=6,\ldots,p+1} = k_i+q
\end{aligned}
\tag{A.6}
$$

where $q=\frac{2}{3}\tilde{k}+\frac{1}{3}\sum_{i=2}^{5}k_i$. A particular case appears for $p=5$ with $\gamma=\theta$, $k_2=k_3=k_4=k_5$, $\tilde{k}=-2k_2$, $k_6=-k_1$, $k_1=\sqrt{3}$, $H=G=1$. This solution corresponds to a six-dimensional self-dual string and provides a nontrivial example of a non-dilatonic SD-brane.

Finally one can consider the case of SDp/SD$(p-6)$-branes. Actually this category only naturally includes the SD6/SD0-brane. From experience with the D6/D0-brane bound states, we expect that the general solution for these four-dimensional dyons will be more complicated than the previous bound states [31] and will require a more elaborate generating solution than given in eq. (2.3) [32]. However, following [33], one can realize that certain special cases should correspond to solutions of the Einstein-Maxwell equations in four spacetime dimensions. Precisely, such an S0-brane in four dimensions was presented in [1] and so that solution can be embedded in type-IIA string theory. Upon lifting this solution to eleven dimensions, it becomes a purely gravitational solution of low energy M-theory equations. A slightly generalized solution has the eleven-dimensional metric:

$$
\begin{aligned}
ds^2 &= \gamma^2\left(-dt^2+t^2(d\chi^2+\sinh^2\chi\,d\lambda^2)\right)+\left(\frac{\alpha\beta}{\gamma}\right)^2dx^2+\\
&\quad+\left(dz+\frac{2\sqrt{2}\sin\theta\,\omega}{\gamma t}dx+2\sqrt{2}\sin\theta\,\omega\cosh\chi\,d\lambda\right)^2+dx^2_{[6]}
\end{aligned}
\tag{A.7}
$$

where as before $\alpha=1+\omega/t$ and $\beta=1-\omega/t$. We have also introduced $\gamma=1+2\cos\theta\,\omega/t+\omega^2/t^2$. When $\sin\theta=0$, $\gamma=\alpha^2$ and the solution reduces to the form given in eq. (2.3). On the other hand, $\cos\theta=0$ yields the M-theory lift of the solution presented in [1]. This concludes our survey of multiply charged S-branes.

B. The IIB SD(-1)-brane

As discussed above, our construction of SDp-brane solutions applied in a straightforward way for $0\le p\le 6$. The cases p=7 and 8 were considered in the discussion section. One might also wonder if there is such an object as an SD(-1)-brane. Recall that the usual

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D(-1)-brane is an instanton carrying a $C^{(0)}$ 'charge' in ten euclidean dimensions. This object is associated with a quantum tunneling process in the type-IIB theory. In contrast, an SD(-1)-brane would be associated with a real-time decay process in ten-dimensional Minkowski space. In common with its instantonic counterpart, however, it should also carry a RR flux of $F^{(9)} = *dC^{(0)}$. While the standard construction presented in section 2 does not apply for $p = -1$, one can construct a candidate type-IIB supergravity solution as follows:

Begin with the eleven-dimensional solution in eq. (2.3) with $p = -1$, i.e., no x^i's. Now dimensionally reducing on z as usual yields:

$$\begin{aligned} ds^2 &= \left(\frac{\beta}{\alpha}\right) \alpha^{\frac{1}{2}} (-dt^2 + t^2 dH_9^2) \\ e^{2\Phi} &= \left(\frac{\beta}{\alpha}\right)^3 \end{aligned} \tag{B.1}$$

where we already have imposed the constraints (2.5). This type-IIA solution only involves NS fields and so is equally valid as a low energy solution of type-IIB supergravity. Now regarding eq. (B.1) as a type-IIB solution, we apply the SL$(2,R)$ transformation [12]

$$\begin{pmatrix} \sin\gamma & -\cos\gamma \\ \cos\gamma & \sin\gamma \end{pmatrix} \tag{B.2}$$

which is chosen so that the asymptotic values of $C^{(0)}$ and $e^{2\Phi}$ remain 0 and 1, respectively. The resulting solution, which is our candidate for the SD(-1)-brane, takes the form

$$\begin{aligned} ds^2 &= F^{\frac{1}{2}} \alpha \beta^{-\frac{1}{2}} (-dt^2 + t^2 dH_9^2) \\ e^{2\Phi} &= F^2 \left(\frac{\beta}{\alpha}\right)^{-3} \\ C^{(0)} &= \sin\gamma\cos\gamma \frac{C}{F} \end{aligned} \tag{B.3}$$

where we have introduced the functions

$$C(t) = 1 - \left(\frac{\beta}{\alpha}\right)^3, \qquad F(t) = \cos^2\gamma + \sin^2\gamma \left(\frac{\beta}{\alpha}\right)^3. \tag{B.4}$$

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